Constraints from the renormalisation of the minimal dark matter model

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Outline

1. Minimal dark matter
2. RGE running
3. Numerical calculation of electroweak mass splitting
   - phenomenological impact
   - how to calculate a mass
   - understanding the origin of the problem

Minimal dark matter

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{2}\overline{\chi}\left(i\not{D} - M\right)\chi$$

where $\not{D}$ is the $SU(2)_L$ covariant derivative.

- Proposed by Cirelli, Fornengo and Strumia [Cirelli et al., 2006, Cirelli and Strumia, 2009].
- Fermionic $SU(2)_L$ quintuplet, χ, with hypercharge $Y = 0$ with interactions:

$$\begin{aligned}\mathcal{L}_{\mathrm{int}} =& g\left(\overline{\chi^{+}}\gamma_\mu\chi^{+} + 2\overline{\chi^{++}}\gamma_\mu\chi^{++}\right)\left(s_w A_\mu + c_w Z_\mu\right) \\ &+ g\left(\sqrt{3}\overline{\chi^{+}}\gamma_\mu\chi^{0} + \sqrt{2}\overline{\chi^{++}}\gamma_\mu\chi^{+}\right)W_\mu^{+} + \mathrm{h.c.}\end{aligned}$$

- **One** new parameter, the MDM tree-level *degenerate* mass, M

Minimal dark matter phenomenology

- Relic density: $M \approx 9.6 \pm 0.2$ TeV
  - by default has a **non-zero mass splitting** $\Delta M = M_{\chi^-} - M_{\chi^0}$ between charged and neutral component

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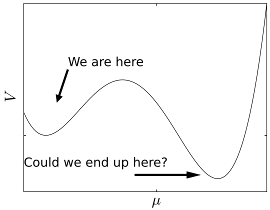
- Vacuum stability
- Perturbitivity

Electroweak vacuum stability

- The standard model Higgs potential is not absolutely stable

- Is nice to make it absolutely stable – and make sure is not absolutely unstable

- We must *renormalise*, $V = V(\mu)$, to quantify this instability



$$V = -m_H^2(\mu)|H|^2 + \lambda(\mu)|H|^4$$

for $\mu \gg M_Z$ we use $\mu \approx \mathcal{O}(h)$

$$V \approx \lambda(h) h^4$$

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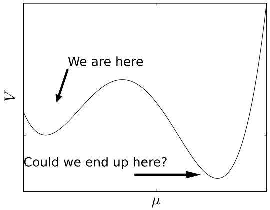

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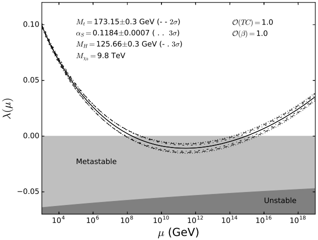
Problem if $\lambda < 0$ as $h \to \infty$

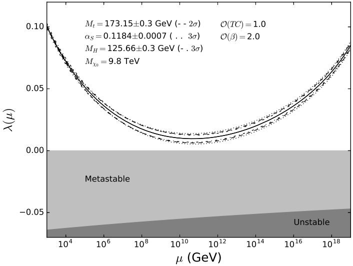
3 types of stability

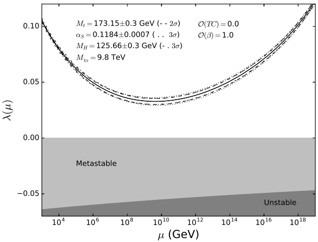
- Stable – one global minimum – Scalar dark matter, Minimal dark matter...
- Meta-stable – but expected time before bubble nucleation $\gg$ age of universe – the standard model
- Unstable – time before bubble nucleation ≈ 0 – hopefully not your new model!

Running of $\lambda(\mu)$ in MDM model



Full 2-loop RGEs solved using FlexibleSUSY with χ integrated in at M_Z





Perturbitivity – Landau poles

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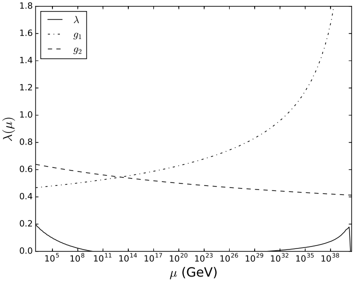
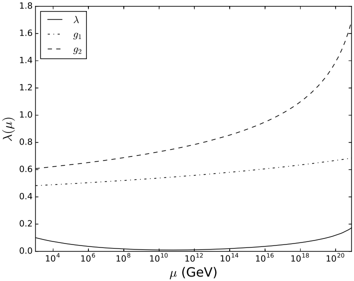
- [Cai et al., 2015] showed "MDM" septuplet has Landau pole at $\approx 10^8$ GeV
- Most recent analysis [Di Luzio et al., 2015] uses partial 2-loop RGEs – MDM Landau pole at 4×10^{21} GeV.
- Using full 2-loop RGEs we find:

SM $\approx 5 \times 10^{40}$ GeV

MDM $\approx 9 \times 10^{21}$ GeV





Spectrum generators

- Spectrum generator – tool which calculates particle masses and couplings given known parameters
- Can solve renormalisation group equations (RGEs)

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- Can solve renormalisation group equations (RGEs)
- e.g. FlexibleSUSY, SPheno, SOFTSUSY
- not only for supersymmetric models

BUT while these are amazing tools we can't always use them "blindly" – the following part of this talk will demonstrate an example of why

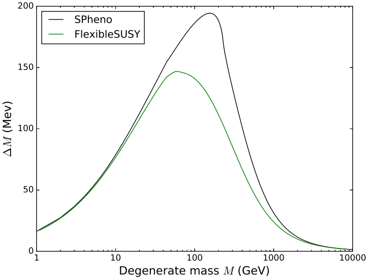
Electroweak multiplet mass splitting

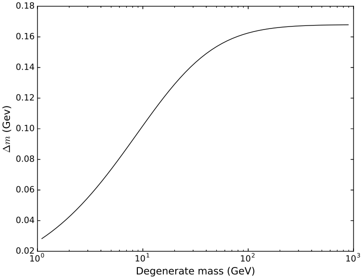
Radiatively induced mass splittings essential for MDM and wino-like dark matter in SUSY

$\Delta M = M_{\chi^-} - M_{\chi^0} \approx 166$ MeV for $M \gg M_Z$
[Cirelli et al., 2006, Ibe et al., 2013, Yamada, 2010, Del Nobile et al., 2010, Ostdiek, 2015]

So any spectrum generator should reproduce this result.

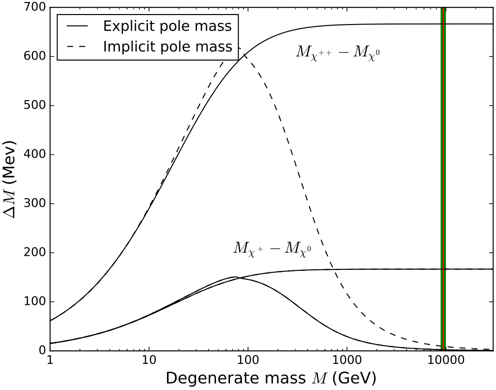
Spectrum generator results





Electroweak multiplet mass splitting

The decoupling between states vanishes for large M depending on how the radiative corrections are calculated

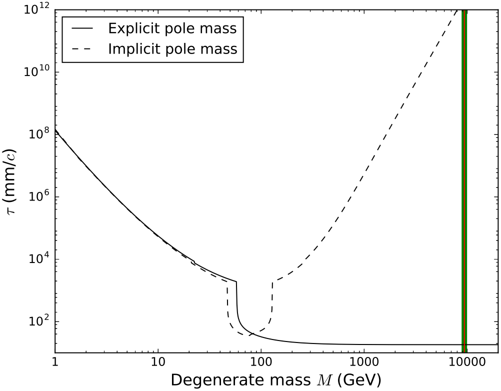


How important is it? – Lifetime of χ^-

The decay width for the electron channel is

$$\Gamma_e = (n^2 - 1)\frac{G_F^2 \Delta M^5}{60\pi^2}$$

and $\Gamma_\mu = 0.12\Gamma_e$



So... how do we calculate a pole mass?

Find the pole of the propagator:

$$\not{p} - M_0 + \Sigma_K(p^2)\not{p} + \Sigma_M(p^2) = 0$$

where $\Sigma_X = \Sigma_X^{(1)} + \Sigma_X^{(2)} + \dots$

So... how do we calculate a pole mass?

setting $p = M_{\text{pole}}$ to be the **pole mass** we have

$$M_{\text{pole}} = \text{Re}\left[\frac{M_0 - \Sigma_M(M_{\text{pole}}^2)}{1 + \Sigma_K(M_{\text{pole}}^2)}\right].$$

now just iterate, easy!

$$M_{\text{pole}} \approx \text{Re}\left[M_0 - \Sigma_M^{(1)}(M_{\text{pole}}^2) - M_0\, \Sigma_K^{(1)}(M_{\text{pole}}^2)\right]$$

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$$\lim_{M\to\infty} \Delta M \to 0\ MeV$$

Explicit pole mass

Approximate!

$$\Sigma_M^{(1)}(M_{\text{pole}}^2) \approx \Sigma_M^{(1)} - 2M_0 \left(M_0\Sigma_K^{(1)} + \Sigma_M^{(1)}\right) \dot{\Sigma}_M^{(1)}\Big|_{p^2=M_0^2}$$

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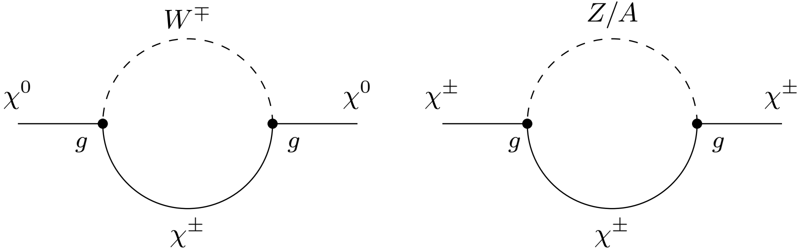
$$\lim_{M\to\infty} \Delta M \to 166 \ MeV$$

How to calculate a pole mass?

So what? They are equivalent expressions to the same order in the gauge coupling. But remember, $\mathcal{O}(M_{\text{pole}}) = 1$ TeV, errors are large, but $\mathcal{O}(\Delta M) = 100$ MeV. Have to look more closely at the self energy functions.

The electroweak multiplet self energy



The electroweak multiplet self energy

Neutral component, W boson only

$$\Sigma_M^{\chi^0}(p^2) = -\frac{3g^2}{2\pi^2}\left(B_0(p^2, M^2, m_W^2) - 1\right)$$
$$\Sigma_K^{\chi^0}(p^2) = -\frac{3g^2}{4\pi^2}\left(B_1(p^2, M^2, m_W^2)\right)$$

Charged component, W, A and Z

$$\Sigma_M^{\chi^-}(p^2) = \frac{-g^2 M}{4\pi^2}\left(s_W^2 B_0(p^2, M^2, 0) + c_W^2 B_0(p^2, M^2, m_Z^2) + 5B_0(p^2, M^2, m_W^2) - 3\right)$$
$$\Sigma_K^{\chi^-}(p^2) = \frac{-g^2 M}{8\pi^2}\left(s_W^2 B_1(p^2, M^2, 0) + c_W^2 B_1(p^2, M^2, m_Z^2) + 5B_1(p^2, M^2, m_W^2)\right)$$

Explicit pole mass result

$$
\begin{aligned}
\Delta M &= M_{\chi^-} - M_{\chi^0} \\
&= \left[M - \Sigma^{\chi^-}(M, M)\right] - \left[M - \Sigma^{\chi^0}(M, M)\right] \\
&= \Sigma^{\chi^0}(M, M) - \Sigma^{\chi^-}(M, M) \\
&\approx M\left(B_0(M, M, m_z) - B_0(M, M, m_w)\right) + \ldots
\end{aligned}
$$

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So we end up with

$$\Delta M \to \frac{\alpha_2}{2}\left(m_W - c_W^2 m_Z\right)$$

What is causing this difference?

$$\Sigma(M_{\text{pole}}^2) = \Sigma(M_{\text{pole}}^2, M^2, m_W^2) \neq \Sigma(M^2, M^2, m_W^2) = \Sigma(M^2)$$

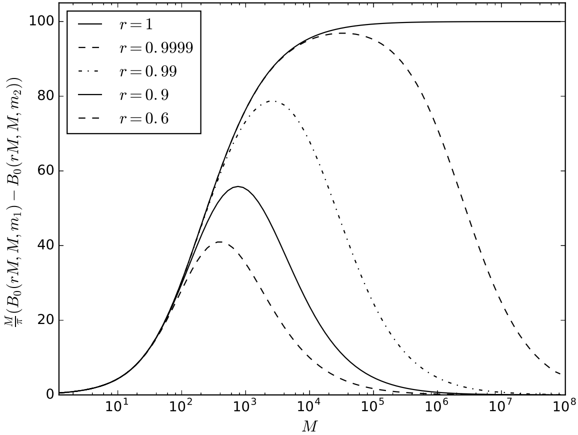
$$\updownarrow$$

$$B_n(M_{\text{pole}}^2, M^2, m_W^2) \neq B_n(M^2, M^2, m_W^2)$$

Some kind of threshold problem in the Passarino-Veltman functions

Difference of Passarino-Veltmann functions

$$\frac{M}{\pi}(B_n(rM, M, m_1) - B_n(rM, M, m_2))$$



So which method is “right”?

- At 1-loop – both are finite and equivalent to the same loop order
- Need to go to 2-loop order
- At 2-loop – iterative method is IR divergent due to photons yet explicit method has cancellation of IR divergences from derivates of 1-loop functions [Ibe et al., 2013]

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$$\Sigma^{(1)}(M_{\text{pole}}^2) \approx \Sigma^{(1)} + C_1\dot{\Sigma}^{(1)} + C_1\dot{\Sigma}^{(1)} + \mathcal{O}(g^6)\Big|_{p^2=M_0^2}$$

$$\Sigma^{(2)}(M_{\text{pole}}^2) \approx \Sigma^{(2)} + \Sigma^{(2)} + \mathcal{O}(g^6)\Big|_{p^2=M_0^2}$$

Summary

- Provided full two-loop results for vacuum stability and perturbativity
- Important problem in the numerical calculation of pole masses – with implications for MDM and Wino dark matter
- Publication in preparation including the results presented here and two-loop self energies

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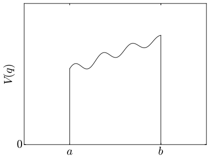
Additional slides

The probability of quantum tunnelling

$$\text{“probability of decay”} \approx A\, e^{-B/\hbar} \approx (T_u \Lambda_B)^4 e^{-S_E}.$$



$$S_E \equiv 2\pi^2 \int_0^\infty \rho^3 \mathrm{d}\rho \left[\frac{1}{2} \left(\frac{\mathrm{d}\phi}{\mathrm{d}\rho} \right)^2 + V \right]$$

where $\rho = \left(\tau^2 + |\vec{x}|^2\right)^{1/2}$ and ϕ satisfies the Euclidean equation of motion

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\rho^2} + \frac{3}{\rho}\frac{\mathrm{d}\phi}{\mathrm{d}\rho} = \frac{\mathrm{d}V}{\mathrm{d}\phi}$$

$\lim_{\rho\to\infty} \phi(\rho) = q_0$, $t \to -\infty$, the global vacuum at $t = 0$ and back to start at $t \to \infty$.

$$S_E = \frac{8\pi^2}{3\,|\lambda|}$$

$$p \approx \left(e^{140} \frac{\Lambda_B}{M_{\text{Planck}}} \right)^4 \exp\left(-\frac{26}{|\lambda_{\text{min}}|} \right)$$

The probability that <number of events> = 0 is

$$\mathcal{L} = \exp\left[-p\right]$$

RGE running in MDM model

At the one-loop level

$$\beta^{(1)}_{g_2,\mathrm{SM}} = -\frac{19}{6} g_2^3 \to \beta^{(1)}_{g_2,\mathrm{MDM}} = \frac{7}{2} g_2^3 \tag{1}$$

and at the two-loop level

$$\begin{aligned}\beta^{(2)}_{g_2,\mathrm{SM}} =& \frac{1}{30} g_2^3 (360 g_3^2 - 45\mathrm{Tr}(Y_d Y_d^\dagger) - 45\mathrm{Tr}(Y_u Y_u^\dagger) \\ & - 15\mathrm{Tr}(Y_e Y_e^\dagger) + 27 g_1^2 + 175 g_2^5)\end{aligned} \tag{2}$$

is modified as (using SARAH to generate RGEs)

$$\beta^{(2)}_{g_2,\mathrm{MDM}} = \beta^{(2)}_{g_2,\mathrm{SM}} + 180\, g_2^5. \tag{3}$$

Consider a scalar field ϕ, for bubble nucleation in flat 4-dimensional spacetime we can take ϕ to be a function of $\rho = \left(\tau^2 + |\vec{x}|^2\right)^{1/2}$ only, as there exists an $O(4)$ symmetry. B is given by the action

$$B = S_E \equiv 2\pi^2 \int_0^\infty \rho^3 \mathrm{d}\rho \left[\frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}\rho}\right)^2 + V\right] \tag{4}$$

for ϕ satisfying the Euclidean equation of motion

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\rho^2} + \frac{3}{\rho}\frac{\mathrm{d}\phi}{\mathrm{d}\rho} = \frac{\mathrm{d}V}{\mathrm{d}\phi} \tag{5}$$

for which the boundary conditions can be combined into the single requirement that $\lim_{\rho\to\infty} \phi(\rho) = q_0$, the solution to (**??**) is known as the bounce solution, as it describes, in Euclidean time, a solution which is in the unstable vacuum at $t \to -\infty$, the global vacuum at $t = 0$ and bounces back to the starting point at $t \to \infty$.

$$h(\rho) = \sqrt{\frac{2}{|\lambda|}} \frac{2R}{\rho^2 + R^2} \tag{6}$$

and from we obtain the action

$$S_E = \frac{8\pi^2}{3\,|\lambda|} \tag{7}$$

$$\Gamma \approx (T_u \Lambda_B)^4 e^{-S_E}. \tag{8}$$

The probability of quantum tunnelling

$$\text{“probability of decay”} \approx A\, e^{-B/\hbar}$$

B – relatively easy to calculate from potential

A – much more difficult but can make good approximations on dimensional grounds

The likelihood function

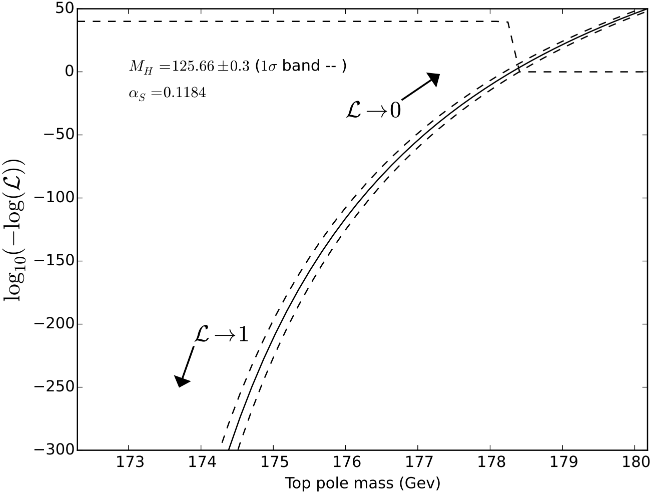
Typically called the "probability", which is interpreted as the expected number of decay events

$$p \approx \left(e^{140} \frac{\Lambda_B}{M_{\text{Planck}}} \right)^4 \exp\left(- \frac{26}{|\lambda_{\text{min}}|} \right)$$

The probability that $<$number of events$> = 0$ is

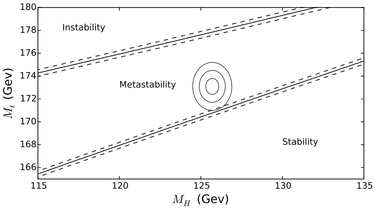
$$\mathcal{L} = \exp\left[-p\right]$$

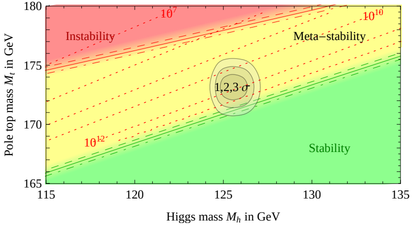
The number of decay events/ log of the likelihood



Stability regions in the SM

Right: Figure from Degrassi, G., Di Vita, S., Elias-Miro et al. 2012





Bound for absolute vacuum stability

$$M_H > 130.4 + 1.4 \left(\frac{M_t - 173.1}{0.7} \right) - 0.6 \left(\frac{\alpha_S - 0.1184}{0.0007} \right)$$